2017 Admission Exam for Shirai Seminar
Faculty of Economics, Keio University
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Microeconomics

There are two agents in an economy. Each consumes two kinds of goods X and Y.

Agent 1's utility function is $U_1=X_1 \cdot Y_1$ where X_1 and Y_1 are amounts of goods X and Y consumed by agent 1.

Similarly, agent 2's utility function is $U_2=2X_2+Y_2$.

Initial endowments of goods X and Y for agent 1 and 2 are 5 units. (Both agents are endowed with 5 units of goods X and Y)

Assume that these two agents buy and sell goods to one another in a perfectly competitive manner; the price of goods X in terms of goods Y is denoted by p (and price of goods Y is 1).

Q1-1. Write down the budget constraint for agent 1 and 2.

Q1-2. Set up utility maximization problem for agent 1 and 2 respectively.

Q1-3. What are the first order conditions of utility maximization for agent 1?

Q1-4. Derive demand function of goods X and Y for agent 1.

Q1-5. What are the first order conditions of utility maximization for agent 2?

Q1-6. Derive demand function of goods X and Y for agent 2.

Q1-7. Draw an offer curve (price-consumption curve) for agent 1 in an Edgeworth box diagram.

Q1-8. Draw an offer curve for agent 2 in an Edgeworth box diagram.

Q1-9. What is the equilibrium condition(s) for exchange between agent 1 and 2 under the assumption of perfect competition?

Q1-10. What is the equilibrium price?

Q1-11. Draw a contract curve (Pareto Efficient consumption schedule) for this economy.

Solution

Q1-1 Agent 1's budget constraint: $pX_1+Y_1 \leq 5p+5$

Agent 2's budget constraint: $pX_2+Y_2 \leq 5p+5$

Q1-2

Maximize $X_1 \cdot Y_1$ subject to agent 1's budget constraint

Maximize $2X_2 + Y_2$

subject to agent 2's budget constraint and $X_2 \geq 0$ and $Y_2 \geq 0$

Q1-3 Set the Lagrangian of agent 1's maximization problem as

$$\mathcal{L}^1 = X_1 \cdot Y_1 + \lambda_1 \cdot (5p + 5 - pX_1 - Y_1).$$

Then the first order conditions are

$$\mathcal{L}^1_{X_1} \equiv Y_1 - \lambda_1 p = 0,$$
$$\mathcal{L}^1_{Y_1} \equiv X_1 - \lambda_1 = 0, \text{ and}$$
$$\mathcal{L}^1_{\lambda_1} \equiv 5(\mathrm{p} + 1) - \mathrm{p}X_1 - Y_1 = 0.$$

Q1-4 We can solve above first order conditions for X_1 and Y_1 in terms of price p. Denote the solution as $X_1(p)$ and $Y_1(p)$;

$$X_1(p) = 5(p + 1)/2p \quad \text{and} \quad Y_1(p) = 5(p + 1)/2.$$

Q1-5 Set the Lagrangian of agent 1's maximization problem as

$$\mathcal{L}^2 = 2X_2 + Y_2 + \lambda_2 \cdot (5p + 5 - pX_2 - Y_2).$$

The Kuhn-Tucker conditions are

$$\mathcal{L}^2_{X_2} \leq 0\,, \ \mathcal{L}^2_{X_2} \cdot X_2 = 0 \ and \ X_2 \geq 0,$$
$$\mathcal{L}^2_{Y_2} \leq 0\,, \ \mathcal{L}^2_{Y_2} \cdot Y_2 = 0 \ and \ Y_2 \geq 0, \text{ and}$$
$$\mathcal{L}^2_{\lambda_2} \equiv 5(\mathrm{p} + 1) - \mathrm{p}X_2 - Y_2 = 0,$$

which are equivalent to

$$2 - \lambda_2 p \leq 0, \ (2 - \lambda_2 p)X_2 = 0 \ and \ X_2 \geq 0,$$
$$1 - \lambda_2 \leq 0, \ (1 - \lambda_2)Y_2 = 0 \ and \ Y_2 \geq 0,$$
$$5(\mathrm{p} + 1) - \mathrm{p}X_2 - Y_2 = 0.$$

Q1-6 We can solve above first order conditions for X_2 and Y_2 in terms of p. Denote the solution as $X_2(p)$ and $Y_2(p)$;

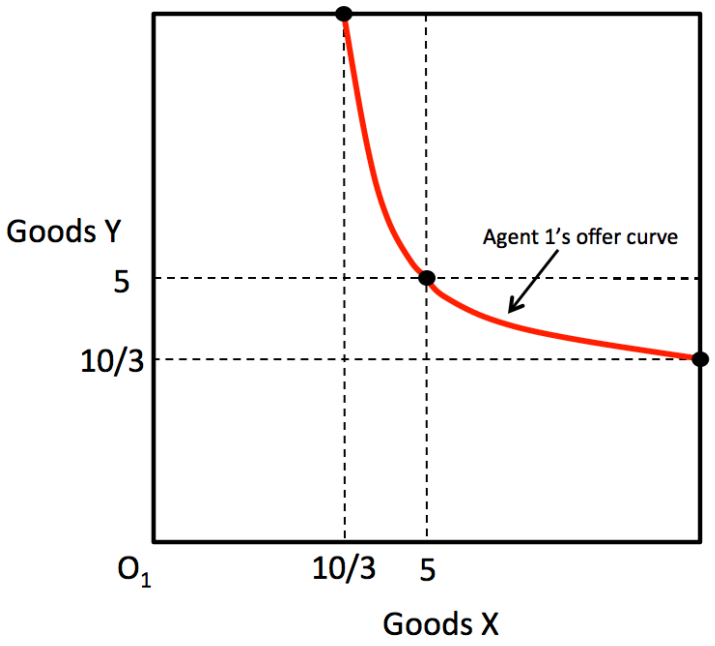
$$X_2(p) = \begin{cases} 0 & if \ \ p > 2 \\ [0, 15/2] & if \ \ p = 2 \\ 5(p+1)/p & if \ \ p < 2 \end{cases}$$

and

$$Y_2(p) = \begin{cases} 5(p+1) & if \ \ p > 2 \\ [0, 15] & if \ \ p = 2 \\ 0 & if \ \ p < 2 \end{cases}$$

where $2X_2(2)+Y_2(2)=15$.

Q1-7 The red curve in a figure below is the agent 1's offer curve.

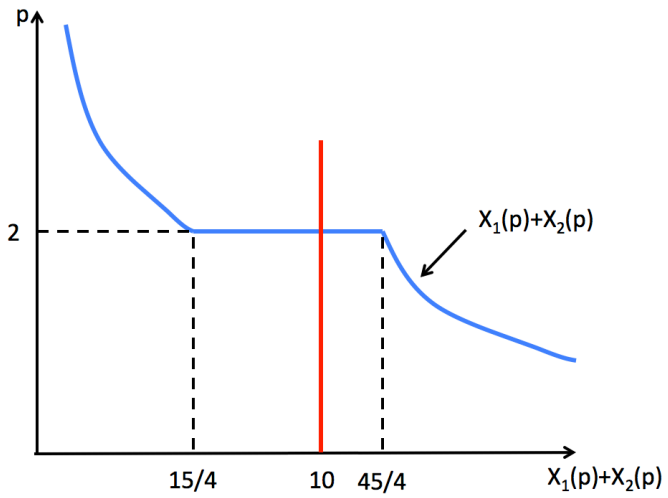


Q1-8. The blue curve in a figure below is the agent 2's offer curve.

Q1-9. The equilibrium condition is,

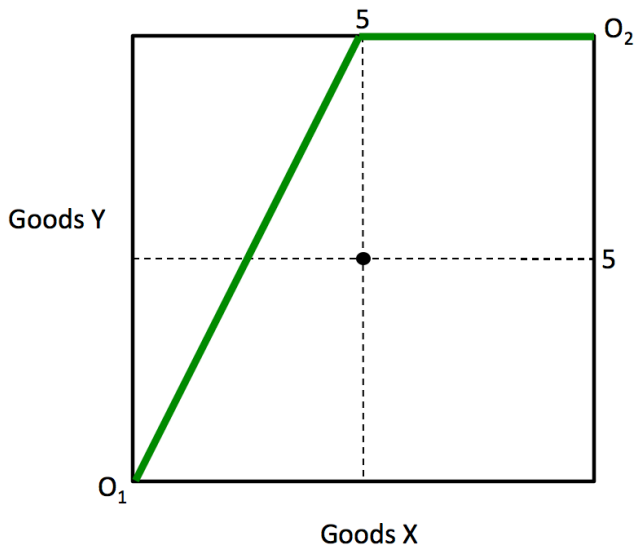
$$X_1(p)+X_2(p)=10. \quad (\text{or } Y_1(p)+Y_2(p)=10)$$

The left hand side of above equation is aggregate demand and the left hand side is aggregate supply. Aggregate demand curve is drawn as a red curve and aggregate supply curve is drawn as a blue line in the figure below.



Q1-10. The equilibrium price is p=2. (The equilibrium consumptions are X_1=15/4, X_2=25/4, Y_1=15/2, and Y_2=5/2)

Q1-11. The green lines are the contract curve for this economy.



Macroeconomics

Read the article “The Age of Secular Stagnation” by L. Summers (*Foreign Affairs* March/April 2016, pp. 2-9) and answer following questions.

Q2-1. Translate the sentence underlined Q2-1 into Japanese.

Q2-2. Explain briefly what “this situation” underlined Q2-2 is?

Q2-3. What is the concept of secular stagnation? Explain.

Q2-4. What is “neutral” interest rate? Explain.

Q2-5. What is the author’s policy recommendation to overcome the problem of secular stagnation?

Explain following terms

Q2-6a. propensity to save

Q2-6b. liquidity trap

Q2-6c. quantitative easing